Ratchet universality in the presence of thermal noise

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We show that directed ratchet transport of a driven overdamped Brownian particle subjected to a spatially periodic and symmetric potential can be reliably controlled by tailoring a biharmonic temporal force, in coherence with the degree-of-symmetry-breaking mechanism. We demonstrate that the effect of finite temperature on the purely deterministic ratchet scenario can be understood as an *effective noise-induced change* of the potential barrier which is in turn controlled by the degree-of-symmetry-breaking mechanism. A possible experimental realization of the present ratchet scenario is proposed in the context of a nanoscale ratchet composed of a pinion and a rack coupled via the lateral Casimir force.

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Directed transport without any net external force, the ratchet effect [1,2], has been an intensely studied interdisciplinary subject over the last few decades owing to its relevance in biology where ratchet mechanisms are found to underlie the working principles of molecular motors [3,4], and to its wide range of potential technological applications including micro- and nano-technologies. Directed ratchet transport (DRT) is today understood to be a result of the interplay of symmetry breaking [5], nonlinearity, and non-equilibrium fluctuations, in which these fluctuations include temporal noise [2], spatial disorder [6], and quenched temporal disorder [7]. In extremely small systems, including many of those occurring in biological and liquid environments as well as many nanoscale devices, DRT is often suitably described by overdamped ratchets, in which inertial effects are negligible in comparison with friction effects. Three of the great diversity of contexts in which overdamped ratchets have been considered are annular Josephson junctions embedded in an inhomogeneous magnetic field [8], synchronization phenomena of coupled oscillators [9], and quasiperiodicity routes in cold atoms [10]. Recently, the dynamics of nanoscale systems composed of one corrugated cylinder (pinion) and one [11] or two [12] corrugated plates (racks) at zero temperature have been studied in the overdamped regime. The pinion and rack(s) have no mechanical contact, but are coupled via the (lateral) Casimir force [13]. Indeed, it has been shown that the lateral Casimir force between corrugated surfaces provides a possibility for frictionless transduction of lateral forces in nanomechanical devices without any physical contact between them [14,15]. More recently, there have been studies of a Casimir-force-driven ratchet in the presence of inertia and finite thermal noise [16], and of a thermal Brownian motor coupled by Casimir interaction in the overdamped regime [17].

In this Letter, we show how ratchet universality [18,19] works subtly in the context of *noisy* overdamped ratchets

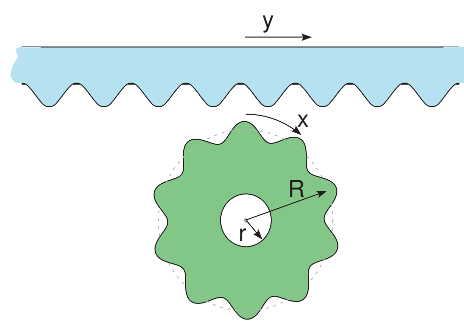

FIG. 1: (Color online) The schematics of the noncontact rack and pinion, with the rack vibrating biharmonically laterally. The rack and pinion have sinusoidal corrugations of the same amplitude and wavelength λ, while r and R are the internal and external radii of the pinion.

by studying a strongly damped pinion at finite temperature that is kept at a distance from a vibrating rack (see Fig. 1). For deterministic ratchets, this has been shown to also be the case for topological solitons [7] and matter-wave solitons [20]. We consider the standard case in which both the pinion and the rack present sinusoidal corrugations with a single wavelength λ, and assume, in contrast with the previously considered cases of uniform [21] and harmonic [22] motion, a biharmonic motion of the rack:

$$y = A\left[\eta\cos\left(\omega t\right) + \left(1-\eta\right)\cos\left(2\omega t+\varphi\right)\right], \qquad (1)$$

where A is an amplitude factor, and the parameters $\eta \in [0,1]$ and φ account for the relative amplitude and initial phase difference of the two harmonics, respectively. The rack and pinion are coupled by the lateral Casimir force between the neighbouring surface areas,

$$F_{lateral} = -F\sin\left[\frac{2\pi}{\lambda}\left(x-y\right)\right], \qquad (2)$$

where F is the amplitude and $x-y$ is the lateral relative displacement. After using $x = R\theta$, where θ is the angle of rotation, one obtains the equation of motion for the pinion:

$$RF\sin\left[\frac{2\pi}{\lambda}\left(x-y\right)\right]+\frac{\zeta}{R}\frac{dx}{dt}=\sqrt{D}\xi\left(t\right), \qquad (3)$$

where ζ is the rotational friction coefficient, $\xi\left(t\right)$ is a Gaussian white noise with zero mean and $\left\langle \xi\left(t\right)\xi\left(t+s\right)\right\rangle = \delta\left(s\right)$, and $D = 2\zeta k_bT$ with k_b and T being the Boltzmann constant and temperature, respectively. After substituting Eq. (1) into Eq. (3) and using the dimensionless variables

$$u\equiv\frac{2\pi}{\lambda}\left(x-y\right), \tau\equiv\frac{2\pi FR^2t}{\lambda\zeta}, \gamma\equiv\frac{A\omega\zeta}{FR^2},$$
$$\Omega\equiv\frac{\lambda\zeta\omega}{2\pi FR^2}, \sigma\equiv\frac{4\pi k_bT}{\lambda F}, \qquad (4)$$

one can rewrite the equation of motion as

$$\dot{u}+\sin u=\sqrt{\sigma}\xi\left(\tau\right)+\gamma F_{bihar}\left(\tau\right), \qquad (5)$$

with $\dot{u} \equiv du/d\tau$ and $F_{bihar}\left(\tau\right) \equiv \eta\sin\left(\Omega\tau\right) + 2\left(1-\eta\right)\sin\left(2\Omega\tau+\varphi\right)$. Thus, Eq. (5) maps the pinion dynamics to the dynamics of a universal model – a Brownian particle moving on a periodic substrate subjected to a biharmonic excitation [23,2]. It is worth noting that, in spite of the abundance of numerical findings, the theoretical understanding of the directed transport phenomena represented by Eq. (5) remains far from being satisfactory [24] even about half a century after the earliest studies [23]. The occurrence of DRT in Eq. (5) implies the breakage of two temporal symmetries: the shift symmetry and the time-reversal symmetry of the biharmonic excitation [2]. For deterministic ratchets subjected to biharmonic forces, it has been shown [18] that there exists a universal force waveform which optimally enhances directed transport by symmetry breaking. Specifically, such a particular waveform has been shown to be unique for both temporal and spatial biharmonic forces. This universal waveform is a direct consequence of the degree-of-symmetry-breaking (DSB) mechanism. In particular, it is possible to consider a quantitative measure of the DSB on which the strength of directed transport by symmetry breaking must depend. This mechanism has led to the unveiling of a criticality scenario for DRT. Indeed, it has been shown that optimal enhancement of DRT is achieved when maximally effective (i.e., *critical*) symmetry breaking occurs, which is in turn a consequence of two reshaping-induced competing effects – the increase of the DSB, and the decrease of the (normalized) maximal transmitted impulse over a half-period – thus implying the existence of a particular force waveform which optimally enhances DRT (see [18] for additional details). Since thermal noise is significant in magnitude and unavoidable at the nanoscale, the following question naturally arises: How does the DSB mechanism work at finite temperatures?

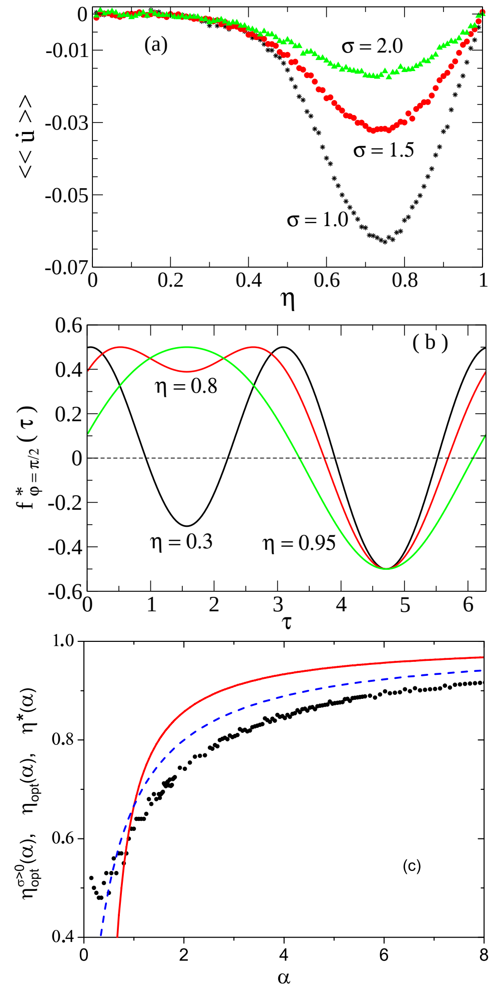

FIG. 2: (Color online) (a) Average velocity $\left\langle\left\langle \dot{u}\right\rangle\right\rangle \equiv \left\langle\left\langle \Lambda du/d\tau'\right\rangle\right\rangle$ [cf. Eqs. (5) and (7)] versus relative amplitude η for $\varphi = \varphi_{opt} \equiv \pi/2, \Omega = 0.08\pi, \gamma = 2$, and three values of the noise intensity. (b) Normalized biharmonic function [Eq. (6)] versus time for $\Omega = 1$ and three values of η. (c) Value of η where the average velocity is maximum, $\eta_{opt}^{\sigma>0}$, versus α [cf. Eq. (5) with $F'_{bihar}\left(\tau\right)$ instead of $F_{bihar}\left(\tau\right)$, see the text] for $\varphi = \varphi_{opt} \equiv \pi/2, \Omega = 0.08\pi, \gamma = 2$, and $\sigma = 1$. Also plotted is the theoretical prediction for the purely deterministic case $\eta_{opt}\left(\alpha\right) \equiv 2\alpha/\left(1+2\alpha\right)$ (dashed line) and the function $\eta^{*}(\alpha)$ (see the text, solid line).

Here, we shall address this important question and provide analytical estimates for the dependence of the DRT on the system's parameters which are in excellent agreement with numerical results. To study numerically the effect of thermal noise ($\sigma > 0$) on the purely deterministic ratchet scenario, we calculated the mean velocity on averaging over different realizations of noise $\left\langle\left\langle \dot{u}\right\rangle\right\rangle$ (cf. Eq. (5)). This corresponds to the mean velocity of the pinion $\left\langle\left\langle \dot{x}\right\rangle\right\rangle = \lambda\left\langle\left\langle \dot{u}\right\rangle\right\rangle/\left(2\pi\right)$. Since Gaussian white noise does not break any relevant symmetry of Eq. (5), and the ratchet universality [18] predicts (for $\sigma = 0$) that the optimal value of the relative amplitude η comes

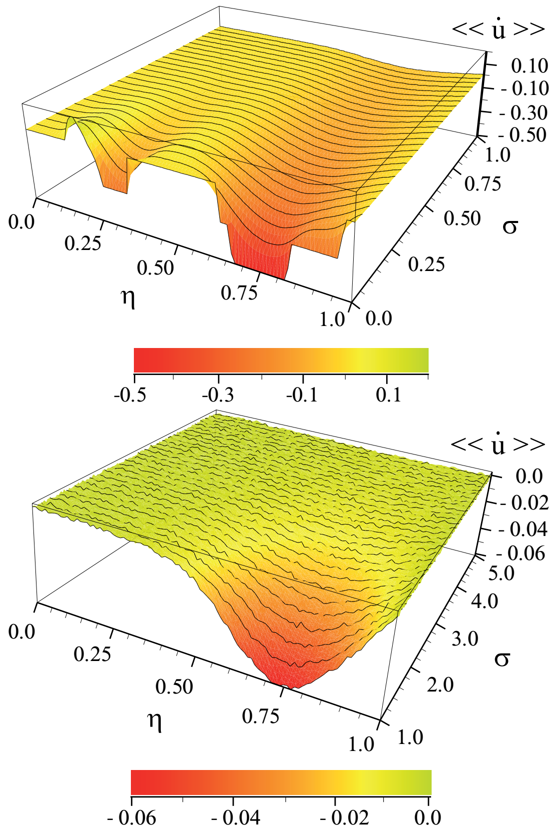

FIG. 3: (Color online) Average velocity $\langle\langle \dot{u} \rangle\rangle$ [cf. Eq. (5)] versus relative amplitude η and noise intensity σ for $\varphi = \varphi_{opt} \equiv \pi/2$, $\Omega = 0.08\pi$, and $\gamma = 2$. The ranges of small and large noise intensities are shown in the top and bottom panels, respectively.

from the condition that the amplitude of $\sin(\Omega\tau)$ must be twice as large as that of $\sin(2\Omega\tau + \varphi)$ in Eq. (5), naïvely, one might expect that the average velocity of the pinion should present, as a function of η, a single maximum at $\eta = \eta_{opt} \equiv 4/5$ when $\varphi \neq 0, \pi$, as in the purely deterministic case. However, our numerical estimates of the η value at which the average velocity is maximum, $\eta_{opt}^{\sigma>0}$, indicate a systematic deviation from $\eta_{opt} \equiv 4/5$: $\Delta\eta \equiv 4/5 - \eta_{opt}^{\sigma>0} > 0$, which is independent of the noise intensity over a significant finite range, as is shown in Figs. 2(a) and 3. To explain this paradox, the following remarks are in order. First, the effect of noise on the DRT strength depends on the amplitude of the biharmonic excitation while keeping the remaining parameters constant. Second, while changing η and φ allows one to control the breakage of the aforementioned relevant symmetries, it also changes the amplitude of the biharmonic excitation. Since the strength of any transport (whether or not induced by symmetry breaking) depends on the amplitude of the driving excitation, one concludes that these two effects of changing η or φ overlap, so that one will find it difficult to distinguish the contribution to transport that is purely due to symmetry breaking, and *hence* to clarify the interplay between noise and symmetry breaking. We shall therefore consider an affine transformation of the biharmonic excitation $F_{bihar}(\tau)$, for the optimal value $\varphi = \varphi_{opt} \equiv \pi/2$ for example [25], to change its image to $[-1/2, 1/2], \forall\eta$, thus making it possible to characterize the genuine effect of noise on the purely deterministic ratchet scenario:

$$f^{*}_{\varphi=\pi/2}(\tau) \equiv \frac{F_{bihar}(\tau) - m}{M - m} - \frac{1}{2}, \tag{6}$$

where $m = m(\eta) \equiv \eta - 2, \forall\eta$, while $M = M(\eta) \equiv \frac{\eta^2 + 32(1-\eta)^2}{16(1-\eta)}$ for $0 \leqslant \eta \leqslant 8/9$ and $M(\eta) \equiv 3\eta - 2$ for $8/9 \leqslant \eta \leqslant 1$ (see Fig. 2(b)). After substituting Eq. (6) into Eq. (5), one straightforwardly obtains

$$\frac{du}{d\tau'} + \frac{1}{\Lambda}\sin u = \gamma W + \gamma f^{*}_{\varphi=\pi/2}(\tau') + \sqrt{\sigma'}\xi(\tau'), \tag{7}$$

where $\Lambda = \Lambda(\eta) \equiv M - m, \tau' = \tau'(\tau, \eta) \equiv \Lambda\tau, W = W(\eta) \equiv (M + m)/(2\Lambda)$, $\Omega' = \Omega'(\Omega, \eta) \equiv \Omega/\Lambda$, and $\sigma' = \sigma'(\sigma, \eta) \equiv \sigma/\Lambda$. Numerical experiments confirmed that the transport properties of Eq. (7) are similar to those of Eq. (5) in the sense that, for both equations, the average velocity $\langle\langle \dot{u} \rangle\rangle \equiv \langle\langle \Lambda du/d\tau' \rangle\rangle$ presents a single extremum at the same value of η for a fixed set of the remaining parameters (see Fig. 2(a)). It is worth noting that the function $\Lambda(\eta)$ is merely the width of the image of $F_{bihar}(\tau)$, i.e., the difference between its maxima and minima as a function of η, and that it presents a single minimum at $\eta = \eta^{*} \equiv 6/7$. Also, the function $W(\eta)$ represents an η-dependent "load" force having a single maximum (in absolute value) at $\eta = \eta_{opt} \equiv 4/5$, while $W(\eta = 0, 1) = 0$, as expected. Note that these particular values of η^{*} and η_{opt} are a direct consequence of the application of ratchet universality to the specific form of the present biharmonic excitation $F_{bihar}(\tau)$. However, to better understand the roots of the present problem, it is convenient to consider the more general form $F'_{bihar}(\tau) \equiv \eta\sin(\Omega\tau) + \alpha(1-\eta)\sin(2\Omega\tau + \varphi)$, with $\alpha > 0$ being a parameter. For this case, one has $\eta^{*} = \eta^{*}(\alpha), \eta_{opt} = \eta_{opt}(\alpha) \equiv 2\alpha/(1 + 2\alpha)$, while the deviation $\Delta\eta(\alpha) \equiv \eta_{opt}(\alpha) - \eta_{opt}^{\sigma>0}(\alpha)$ suggests a certain correlation between $\eta_{opt}(\alpha)$ and $\eta_{opt}^{\sigma>0}(\alpha)$ over a wide range of α values from $\alpha \simeq 2$ (see Fig. 2(c)). Together, these results therefore allow one to draw the following conclusions from Eq. (7).

First, the aforementioned twofold transport effect of a biharmonic excitation $F_{bihar}(\tau)$, as η varies from 0 to 1, may be decoupled into two terms: a constant excitation, W, and a biharmonic excitation, $f^{*}_{\varphi=\pi/2}(\tau')$, having an amplitude which is *independent* of η. The relevant observation is that both excitations yield a maximum strength of transport at $\eta = \eta_{opt} \equiv 4/5$. Therefore, replacing $F_{bihar}(\tau)$ with $f^{*}_{\varphi=\pi/2}(\tau)$ in Eq. (5),

$$\dot{u} + \sin u = \sqrt{\sigma}\xi(\tau) + \gamma f^{*}_{\varphi=\pi/2}(\tau), \tag{8}$$

should yield a maximum average velocity at $\eta = \eta_{opt} \equiv 4/5$, as is indeed confirmed by numerical experiments (see

Fig. 4). Thus, we propose for the system (8) the following scaling for the average velocity:

$$\langle\langle\dot{u}\rangle\rangle \sim CW(\eta), \tag{9}$$

where C is a fitting constant that depends on the remaining system parameters. Furthermore, for sufficiently high temperature (i.e., sufficiently far from the "steps" regime occurring at $\sigma = 0$, see Fig. 3) and driving amplitude (i.e., in the absence of stochastic resonance effects), *exact* agreement between numerical results and scaling (9) is expected over the complete range of η values (see Fig. 4), while the scaling (9) remains valid over a wide range of frequencies (data not shown).

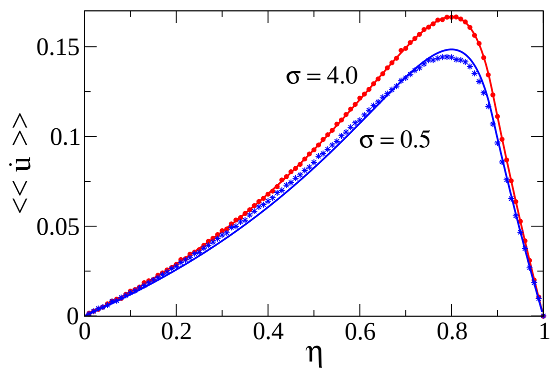

FIG. 4: (Color online) Average velocity $\langle\langle\dot{u}\rangle\rangle$ [cf. Eq. (8)] versus relative amplitude η for $\Omega = 0.08\pi, \gamma = 2$, and two values of the noise intensity. Also plotted is the scaling law (9) for $C = 1.00017$ ($\sigma = 4$) and $C = 0.89072$ ($\sigma = 0.5$) [cf. Eq. (9); solid lines].

And second, alternatively to the case discussed in the first conclusion, *only* rescaling the temperature with the width Λ in Eq. (5) in accordance with Eq. (7),

$$\dot{u} + \sin u = \sqrt{\Lambda\sigma}\xi(\tau) + \gamma F_{bihar}(\tau), \tag{10}$$

should also yield a maximum average velocity at $\eta = \eta_{opt} \equiv 4/5$. Numerical experiments also confirmed this prediction, as in the illustrative examples shown in Fig. 5(a). Thus, it is only after rescaling $\sigma \to \Lambda\sigma$ in Eq. (5) that one recovers the purely deterministic ratchet scenario, which was an unanticipated result. To make the comparison between Eqs. (5) and (10) clearer, let us first transform Eq. (10) into the equation

$$\begin{aligned} \dot{u} + \frac{1}{\Lambda}\sin u &= \sqrt{\sigma}\xi(\tau) \\ &+ \gamma'\left[\eta\sin(\Omega'\tau) + 2(1-\eta)\sin(2\Omega'\tau + \varphi)\right] \end{aligned} \tag{11}$$

by rescaling the time, $\tau \to \Lambda\tau$, and where $\gamma' \equiv \gamma/\Lambda, \Omega' \equiv \Omega/\Lambda$. Since the transport properties of Eq. (11) are similar to those of Eq. (10) in the sense that their respective average velocities present a single extremum at the same value of η for a fixed set of the remaining parameters, and that such an optimal value, $\eta_{opt} \equiv 4/5$, is independent

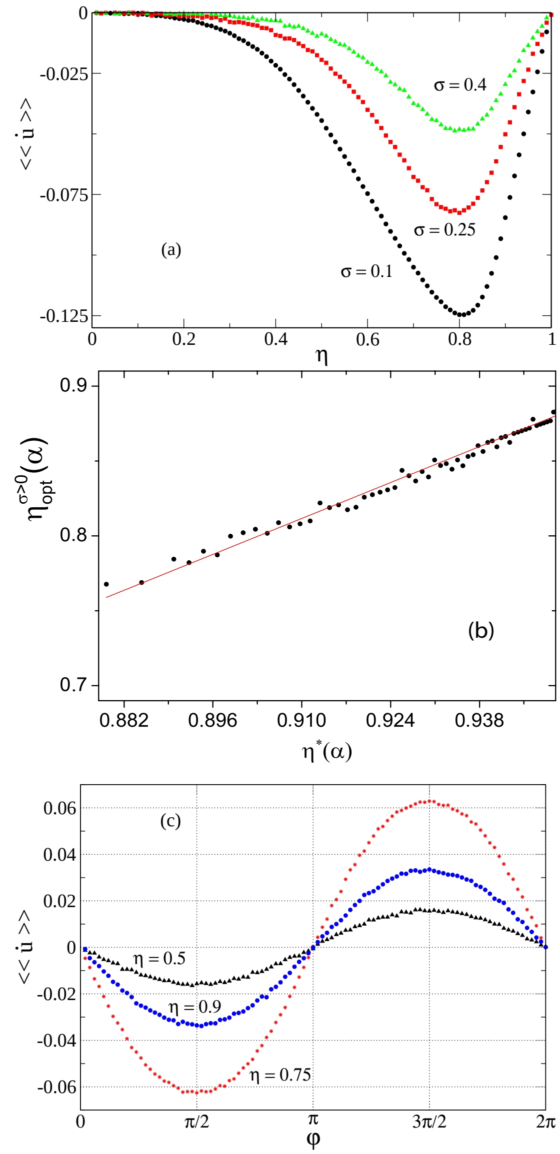

FIG. 5: (Color online) (a) Average velocity $\langle\langle\dot{u}\rangle\rangle$ [cf. Eq. (10)] versus relative amplitude η for $\Omega = 0.08\pi, \gamma = 2$, and three values of the noise intensity. (b) Value of η where the average velocity is maximum, $\eta_{opt}^{\sigma>0}(\alpha)$, versus function $\eta^*(\alpha)$ [cf. Eq. (5) with $F'_{bihar}(\tau)$ instead of $F_{bihar}(\tau)$, see the text] and linear fit (12) (solid line) over the range $2.3 \leqslant \alpha \leqslant 7.2$ for $\varphi = \varphi_{opt} \equiv \pi/2, \Omega = 0.08\pi, \gamma = 2$, and $\sigma = 1$. (c) Average velocity $\langle\langle\dot{u}\rangle\rangle$ [cf. Eq. (5)] versus initial phase difference φ for $\Omega = 0.08\pi, \gamma = 2, \sigma = 1$, and three values of η.

of the particular values of the amplitude and the frequency of the biharmonic excitation [18], one concludes from the comparison of Eqs. (5) and (11) that the effect of thermal noise on the purely deterministic ratchet scenario can be understood as an *effective noise-induced change* of the potential barrier ($d = d(\eta) \equiv \Lambda(\eta)$) which is in turn controlled by the degree-of-symmetry-breaking mechanism through the function $\Lambda(\eta)$. Recall that the average velocity $\langle\langle\dot{u}\rangle\rangle$ exhibits, as a function of the po-

tential barrier d, a single maximum due to the thermal interwell activation (TIA) mechanism and the limiting behaviours $\lim_{d\to 0,\infty}\langle\langle\dot{u}\rangle\rangle = 0$ [24]. Obviously, the same scenario holds when $F_{bihar}(\tau)$ is replaced with $F'_{bihar}(\tau)$, which allows one to understand the behaviour of the deviation $\Delta\eta(\alpha) \equiv \eta_{opt}(\alpha) - \eta_{opt}^{\sigma>0}(\alpha)$ as α is changed (cf. Fig. 2(c)). Clearly, one can distinguish three *regimes*. Over the range $0 < \alpha \lesssim 1/2$, the TIA mechanism dominates over the DSB mechanism. Indeed, the contribution of the DSB mechanism to directed transport becomes ever smaller as $\alpha \to 0$ because $\eta_{opt}(\alpha \to 0) \to 0$, and hence the (whole) amplitude of the biharmonic excitation for which the DRT strength is maximum also becomes ever smaller as $\alpha \to 0$, while the contribution of the TIA mechanism remains significant over the entire range $0 < \alpha \lesssim 1/2$. Then one observes a transition regime over the range $1/2 \lesssim \alpha \lesssim 2$ as the effect of the DSB mechanism strengthens, which is manifest in the existence of a narrow range of α values in which $\Delta\eta(\alpha) \approx 0$ (see Fig. 2(c)). Finally, for the range $\alpha \gtrsim 2$, the contributions of the DSB and TIA mechanisms are correlated in the sense of the aforementioned effective noise-induced change of the potential barrier. This means that $\eta_{opt}^{\sigma>0}(\alpha)$ is expected to be proportional to the value of the relative amplitude where the effective noise-induced potential barrier presents a minimum:

$$\eta_{opt}^{\sigma>0}(\alpha) \sim A\eta^{*}(\alpha) + B, \tag{12}$$

with $A \simeq 1.71, B \simeq -0.744$ being constants that are *independent* of the remaining system parameters. The general scaling (12) is confirmed by numerical experiments (see Fig. 5(b)). Also, with regard to the dependence of the DRT strength on the initial phase difference, numerical results confirmed the scaling $\langle\langle\dot{u}\rangle\rangle \sim C' \sin\varphi$, where C' is a fitting constant that depends on the remaining system parameters, in accordance with ratchet universality [18,19] (see Fig. 5(c)).

In summary, we have explained the interplay between thermal noise and symmetry breaking in the ratchet transport of a Brownian particle moving on a periodic substrate subjected to a temporal biharmonic excitation, in coherence with the degree-of-symmetry-breaking mechanism. We have discussed how the present findings can be readily tested experimentally by means of a strongly damped rack-pinion ratchet at finite temperature, where the rack undergoes a biharmonic lateral motion. The general ratchet scenario presented in this Letter provides a theoretical framework for the optimal control of the dynamics of nanomechanical ratchets in future applications.

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[25] The analysis is quite similar for any other value of the initial phase difference φ. In particular, for the other optimal value, $\varphi_{opt} \equiv 3\pi/2$, one has DRT with the same strength but opposite direction to that corresponding to $\varphi_{opt} \equiv \pi/2$ for each value of $\eta \in]0,1[$ (cf. Ref. [18]).